Calculus I
UNIT 2: Limits & Discontinuity

Name: _____
Date: _____

# UNIT 2 REVIEW

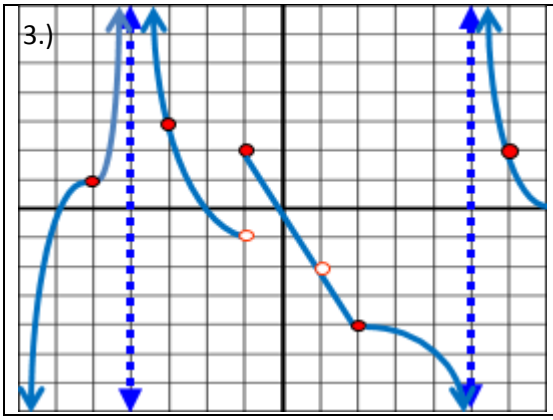
## RATIONAL FUNCTIONS

1.) $f(x) = \frac{(9x^2 - x)}{x^3 - 9x}$ Hole: _____ VA: _____ HA: _____ SA: _____

2.) $f(x) = \frac{3x^2 - 13x - 10}{x^2 - 4x - 5}$ Hole: _____ VA: _____ HA: _____ SA: _____

## FINDING LIMITS GRAPHICALLY

3.)



$\lim_{x \to 5^-} f(x) =$ _____ $\lim_{x \to -4^+} f(x) =$ _____ $f(-1) =$ _____

$\lim_{x \to -1} f(x) =$ _____ $\lim_{x \to 1} f(x) =$ _____ $f(6) =$ _____

$\lim_{x \to -1^-} f(x) =$ _____ $\lim_{x \to 5} f(x) =$ _____ $f(-4) =$ _____

4.) For which value(s) of $x$ does the limit not exist? _____

5.) For which value(s) of $x$ is the graph discontinuous? _____

## FINDING LIMITS ALGEBRAICALLY

6.) $\lim_{x \to 1} \frac{x+7}{x+1} =$

7.) $\lim_{x \to -3} \frac{3x+9}{x^2-9} =$

8.) $\lim_{x \to 3} \frac{3x+9}{x^2-9} =$

9.) $\lim_{x \to \infty} \frac{12x^3 + 10x^2 - 5x}{30x^3} =$

10.) $\lim_{x\to-\infty} \dfrac{x^2-3}{x^3+9} =$

11.) $\lim_{x\to-2} \dfrac{-x^3}{2x-4} =$

12.) $\lim_{x\to4^-} \dfrac{2x+18}{x^2+5x-36} =$

13.) $\lim_{x\to\infty} \dfrac{-10x^4+20x^3}{15x^3} =$

14.) $\lim_{x\to4} \dfrac{x^3-4x^2+6x-24}{x^2+3x-28} =$

15.) $\lim_{x\to1^+} \dfrac{2x^2-x-3}{2x^2-5x+3} =$

**FINDING LIMITS USING THE GRAPHING CALCULATOR**

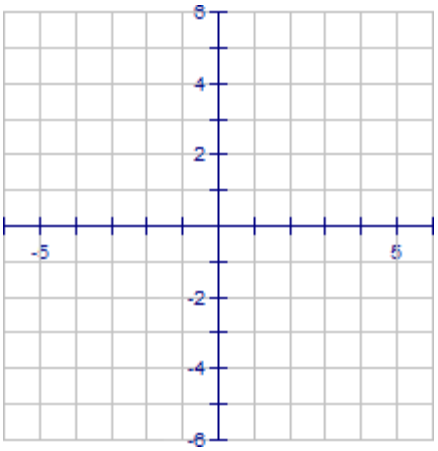
16.) $\lim_{x\to5^+} \dfrac{|x-5|}{x-5} =$

17.) $\lim_{x\to0} = \dfrac{\cos x}{x}$

18.) $\lim_{x\to-3} = \dfrac{x^3+27}{-x-3}$

**CONTINUITY & PIECEWISE FUNCTIONS**

$$f(x) = \begin{cases} -x^2+1, & x<0 \\ 2|x-1|, & x\geq 0 \end{cases}$$




19.) Graph the piecewise function to the right.

20.) Determine the limit from the left-side and the right side.

21.) Determine if the limit exists at the indicated $x$-value.

22.) State whether the function is continuous or discontinuous at the indicated $x$-value.

23.) If it is discontinuous, state what type of discontinuity occurs at the $x$-value.

# UNIT 2 TEST STUDY GUIDE: LIMITS & DISCONTINUITY

Section1 — Finding Discontinuity
- ✓ Point (Hole)
- ✓ Infinite (Asymptotes-VA/HA/SA)

Section 2 — Determining Limits from a Graph
- ✓ Left-Side/Right-Side Limits

Section 3 — Determining Limits
- ✓ Direct Substitution
- ✓ Algebraic Simplification followed by direct substitution

Section 4 — Infinite Limits (x → c, where a VA lies at c!)

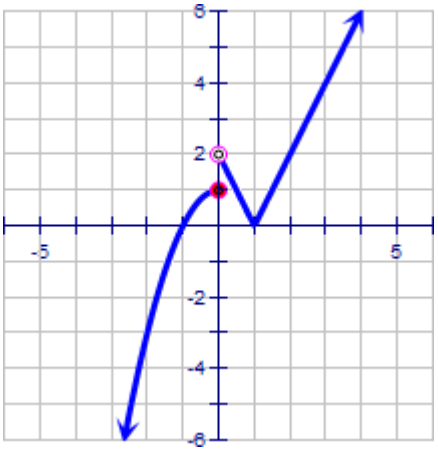
Section 5 — Limits at Infinity (x → $\infty$) OR (x → $-\infty$)

Section 6 — Other Methods for determining Limits (Graphing Calc)
- ✓ Table of Values
- ✓ Use of a Graph

Section 7 — Determining Continuity
- ✓ $f(c)$ exists
- ✓ $\lim_{x\to c} f(x)$ exists
- ✓ $f(c) = \lim_{x\to c} f(x)$

Types of Discontinuity
- ✓ Point (hole)
- ✓ Infinite (vertical asymptote)
- ✓ Jump
- ✓ Non-existence (use interval notation)

Section 8 — Piecewise Functions




# SOLUTIONS

1.) Hole: $\left(0,\frac{1}{9}\right)$ VA: $x = \pm 3$ HA: $y = 0$ SA: None

2.) Hole: $\left(5,\frac{17}{6}\right)$ VA: $x = -1$ HA: $y = 3$ SA: None

3.) $\lim_{x\to 5^-} f(x) = -\infty$ $\lim_{x\to -4^+} f(x) = \infty$ $f(-1) = 2$

$\lim_{x\to -1} f(x) = DNE$ $\lim_{x\to 1} f(x) = -2$ $f(6) = 2$

$\lim_{x\to -1^-} f(x) = -1$ $\lim_{x\to 5} f(x) = DNE$ $f(-4) = \emptyset$

4.) $x = -1, 5$

5.) $x = -4, -1, 1, 5$

6.) 4 7.) $-\frac{1}{2}$ 8.) $DNE$ 9.) $\frac{2}{5}$

10.) 0 11.) $-1$ 12.) $-\infty$ 13.) $-\infty$

14.) 2 15.) $\infty$ 16.) 1 17.) $DNE$

18.) $-27$ 19.) See graph above 20.) $\lim_{x\to 0^-} f(x) = 1$ $\lim_{x\to 0^+} f(x) = 2$

21.) $\lim_{x\to 0} f(x) = DNE$ 22.) Discontinuous at $x = 0$ 23.) Jump Discontinuity